# Surface Impedance for Electromagnetic Fields Over a Dielectric-Coated Circular Cylinder

Andrés G. Aguilar*, Zvonimir Sipus†, and Manuel Sierra-Pérez*.

* Signals, Systems and Radiocommunications Department, ETSI Telecomunicacion,
Technical University of Madrid, Spain. andresg@gr.ssr.upm.es
† Faculty of Electrical Engineering and Computing, University of Zagreb. Croatia. zvonimir.sipus@fer.hr

For small or medium size conformal array antennas in terms of the wave length, modal solutions in spectral domain for mutual coupling analysis are convenient for canonical shapes such as circular cylinder [1] or sphere [2], but as the antenna dimensions increase a large number of terms are necessary. For large structures the uniform theory of diffraction (UTD) is commonly used to solve this problem for canonical and arbitrarily convex shaped perfect electric conductor (PEC) surfaces [3]. A UTD solution for mutual coupling on an impedance cylinder has been introduced in [4], [5] but using a constant surface impedance.

The impedance boundary condition (IBC) can approximate the coated or non-coated PEC cylinder problem by performing a proper surface impedance characterization. In this work, the surface impedance is derived to calculate the surface fields on a dielectric-coated grounded circular cylinder. The geometry is shown in Fig. 1, a metallic circular cylinder of radius $a$ covered by a dielectric layer of thickness $d$, where the source point $P'$ and the observation point $P$ are assumed to be far away. The UTD based asymptotic Green's functions with IBC are modified to include the surface impedance dependence on the geometrical parameters of the rays upon the cylinder.

The eigenfunction solution is calculated by enforcing the boundary conditions over the cylinder surface, where the IBC is

$$\left[\begin{array}{c} \vec{E}_\phi \\ \vec{H}_\phi \end{array}\right] = \left[\begin{array}{cc} -Z_s^e & 0 \\ 0 & Y_s^m \end{array}\right] \left[\begin{array}{c} \vec{H}_z \\ \vec{E}_z \end{array}\right]\Bigg|_{\rho=b} \tag{1}$$

where $Y_s^m$ and $Z_s^e$ are the TM/TE surface admittance/impedance, which define the normalized TM/TE surface admittance/impedance $\Lambda_s^m = \frac{Y_s^m}{Y_0}$ and $\Lambda_s^e = \frac{Z_s^e}{Z_0}$, respectively, with $Z_0 = \frac{1}{Y_0}$ the characteristic impedance of free space. If a constant surface impedance $Z_s$ is considered then

$$\Lambda_s^e = (\Lambda_s^m)^{-1} \equiv \Lambda_s = \frac{Z_s}{Z_0} \tag{2}$$

When $\Lambda_s \to 0$ the PEC case is recovered.

The surface field whether for the impedance or the coated cylinder can be expressed in terms of

$$q_{m,e}(n,k_z) = -j\,\frac{k_{\rho 0}}{k_0}\Lambda_s^{m,e}(n,k_z) \tag{3a}$$

$$\tilde{q}_e^{z\phi}(n,k_z) = -j\,\frac{k_{\rho 0}}{k_0}\tilde{\Lambda}_s^{e,z\phi}(n,k_z) \tag{3b}$$

$$\tilde{q}_e^{\phi\phi}(n,k_z) = -j\,\frac{k_{\rho 0}}{k_0}\tilde{\Lambda}_s^{e,\phi\phi}(n,k_z) \tag{3c}$$

$$q_c^{ibc}(n,k_z) = -j\,\frac{nk_z}{k_0 k_{\rho 0} b} \tag{3d}$$

$$q_c^{coated}(n,k_z) = -j\,\frac{nk_z}{k_0 k_{\rho 0} b}\,\frac{k_0^2(\mu_r\varepsilon_r - 1)}{k_{\rho 1}^2} \tag{3e}$$

Differences on the surface fields for the impedance and the metallic coated circular cylinder are in these $q_{m,e}$ and $q_c$ parameters, except for some changes in the TE surface impedance depending on the field and source orientation for the coated case. For the impedance cylinder $q_m$ and $q_e$ represent the surface impedance dependency for the TM and the TE modes, respectively. When the ray angle is $\alpha = 0^o$ then $q_c = 0$, and when $\alpha \neq 0^o$ both set of modes are coupled through such a $q_c$ term. Thus the effective surface impedance must change with the ray angle $\alpha$ and depend on the wave numbers $n$ and $k_z$. By making a comparison between the coated and the impedance cylinder problems the normalized surface admittance/impedance can be written in terms of Bessel functions as

$$\Lambda_s^m(n,k_z) = -j\,\frac{\varepsilon_r k_0}{k_{\rho 1}}\,\frac{J_n(k_{\rho 1}a)Y_n'(k_{\rho 1}b) - J_n'(k_{\rho 1}b)Y_n(k_{\rho 1}a)}{J_n(k_{\rho 1}a)Y_n(k_{\rho 1}b) - J_n(k_{\rho 1}b)Y_n(k_{\rho 1}a)} \tag{4a}$$

$$\Lambda_s^e(n,k_z) = -j\,\frac{\mu_r k_0}{k_{\rho 1}}\,\frac{J_n'(k_{\rho 1}a)Y_n'(k_{\rho 1}b) - J_n'(k_{\rho 1}b)Y_n'(k_{\rho 1}a)}{J_n'(k_{\rho 1}a)Y_n(k_{\rho 1}b) - J_n(k_{\rho 1}b)Y_n'(k_{\rho 1}a)} \tag{4b}$$

For the $z\phi'$ and $\phi\phi'$ field and source components the TE surface impedance must be different, so

$$\tilde{\Lambda}_s^{e,z\phi}(n,k_z) = j\,\frac{k_{\rho 0}}{k_0(\mu_r\varepsilon_r - 1)}\left(R_n + \frac{k_{\rho 1}^2}{k_{\rho 0}^2}q_e(n,k_z)\right) \tag{5a}$$

$$\tilde{\Lambda}_s^{e,\phi\phi}(n,k_z) = j\frac{k_{\rho 0}(k_0^2-k_z^2)}{k_0^3(\mu_r\varepsilon_r-1)^2}\cdot$$
$$\cdot\left(R_n + \frac{k_{\rho 1}^4}{k_{\rho 0}^2}\frac{1}{k_0^2-k_z^2}q_e(n,k_z)\right) \quad (5b)$$

where $R_n = \frac{H_n^{(2)'}(k_{\rho 0}b)}{H_n^{(2)}(k_{\rho 0}b)}$, and $H_n^{(2)}(z)$ is the Hankel function of second kind. If the surface impedance $Z_s$ is constant then $\tilde{\Lambda}_s^{e,z\phi} = \tilde{\Lambda}_s^{e,\phi\phi} = \Lambda_s$ as in (2).

Thus, the surface fields in spectral domain for an impedance and a coated circular cylinder are given by

$$\tilde{G}_{zz}(n,k_z) = jZ_0\frac{k_{\rho 0}}{k_0}\frac{(R_n+q_e)}{D_n} \quad (6a)$$

$$\tilde{G}_{z\phi}(n,k_z) = \tilde{G}_{\phi z}(n,k_z) = Z_0\frac{q_c\tilde{q}_e^{z\phi}}{D_n} \quad (6b)$$

$$\tilde{G}_{\phi\phi}(n,k_z) = -jZ_0\frac{k_0}{k_{\rho 0}}\frac{q_eR_n(R_n+q_m)+q_c^2\tilde{q}_e^{\phi\phi}}{D_n} \quad (6c)$$

with $D_n = (R_n+q_e)(R_n+q_m)+q_c^2$. A UTD asymptotic expansion is applied to these fields, considering now the azimuthal wave number $n$ and the axial wave number $k_z$ dependancy of the surface impedance.

To simplify equations (4), two-term Debye's asymptotic formulas are applied and a Taylor Series expansion around $1/b = 0$ is performed, where only the first two terms are retained [6], which is valid for a thin dielectric coating, at least, for $d < 0.25\lambda_0$. Above this threshold, Olver's uniform formulation can be applied. For equations in (5) the same approximation can be used for $q_e$ and Watson approximations for Hankel functions can be applied.

Fig. 2 show the mutual impedance between two thin dipoles located over the surface of a grounded dielectric-coated cylinder along the first geodesic ray, comparing the UTD with IBC with the corresponding eigenfunction solution. Electric current modes are defined by sinusoidal along the direction of the current and by constant along the directions perpendicular to the current. Only shortest ray contribution is necessary for this geometry between source and observation points. A very good agreement is met for observation points further than $1\lambda_0$.

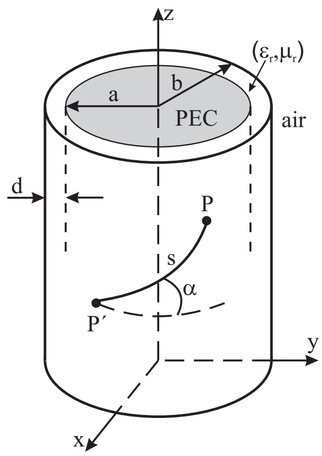


Fig. 1. Surface ray path over an infinitely long dielectric-coated PEC circular cylinder.

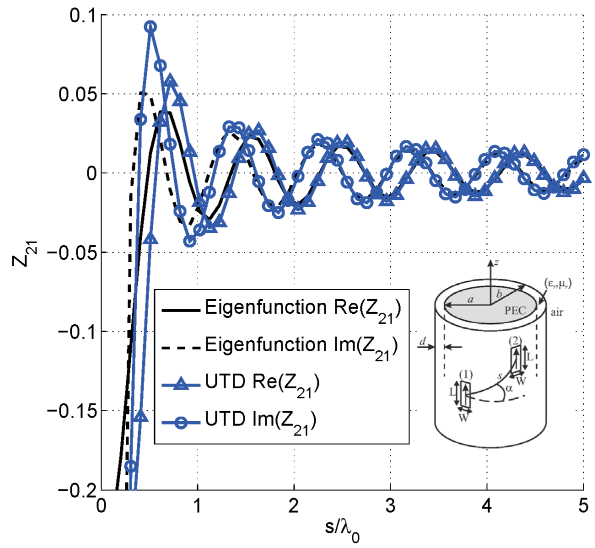


Fig. 2. Mutual impedance for $zz'$ component, with $a = 3\lambda_0$, $\varepsilon_r = 3.25$, $d = 0.06\lambda_0$, $W = 0.02\lambda_0$, $L = 0.05\lambda_0$ and $\alpha = 55^{\circ}$.


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